# EFT Unit 1

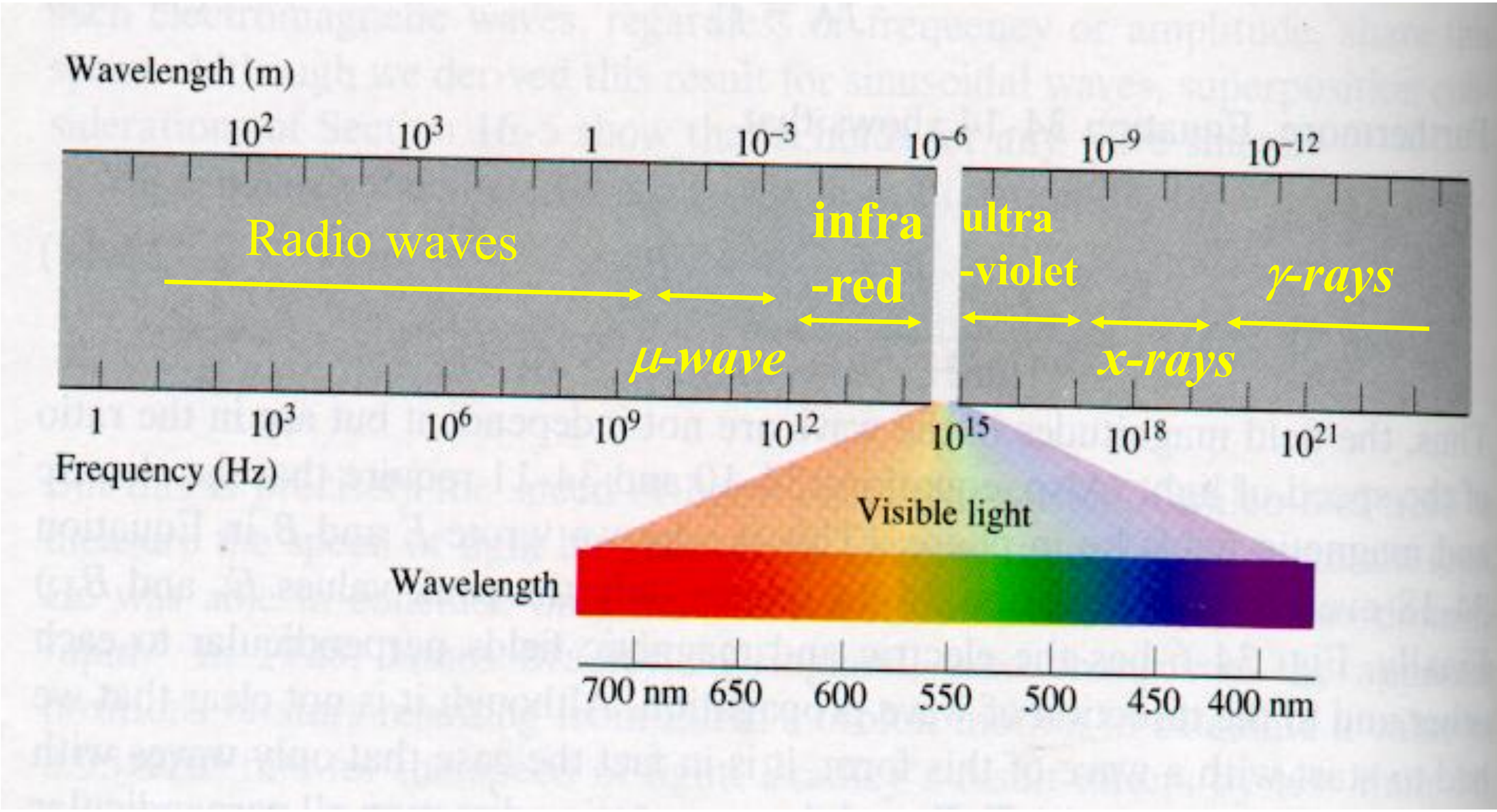
## The Laws of Electromagnetism

## Maxwell’s Equations

## Displacement Current

## Electromagnetic Radiation

# The Electromagnetic Spectrum



# The Equations of Electromagnetism
# (at this point …)

**Gauss' Law for Electrostatics** $\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0}$

**Gauss' Law for Magnetism** $\oint \underline{B} \bullet \underline{dA} = 0$

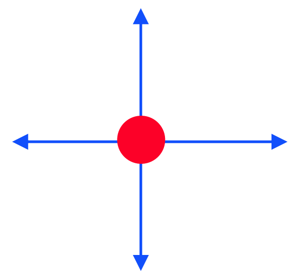
**Faraday's Law of Induction** $\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$

**Ampere's Law** $\oint \underline{B} \bullet \underline{dl} = \mu_0 I$

# The Equations of Electromagnetism

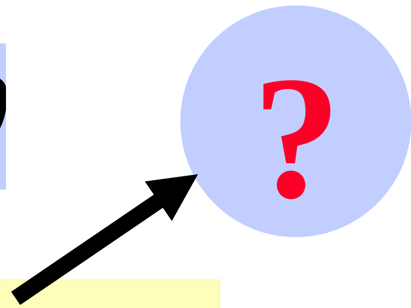
**Gauss's Laws**

***..monopole..***



**1** $$\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0}$$

**2** $$\oint \underline{B} \bullet \underline{dA} = 0$$




***...there's no magnetic monopole....!!***

# The Equations of Electromagnetism

**Faraday's Law**

**3** $$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

***.. if you change a magnetic field you induce an electric field.........***

**Ampere's Law**

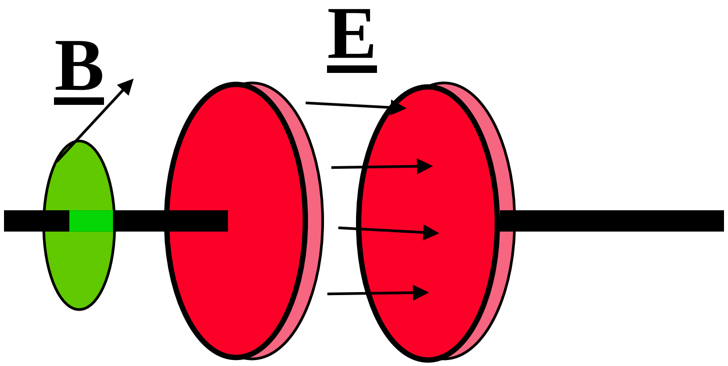
**4** $$\oint \underline{B} \bullet \underline{dl} = \mu_0 I$$

***.......is the reverse true..?***

# ...lets take a look at charge flowing into a capacitor...

**...when we derived Ampere's Law we assumed constant current...**

$$\boxed{\oint \underline{B} \bullet \underline{dl} = \mu_0 I}$$



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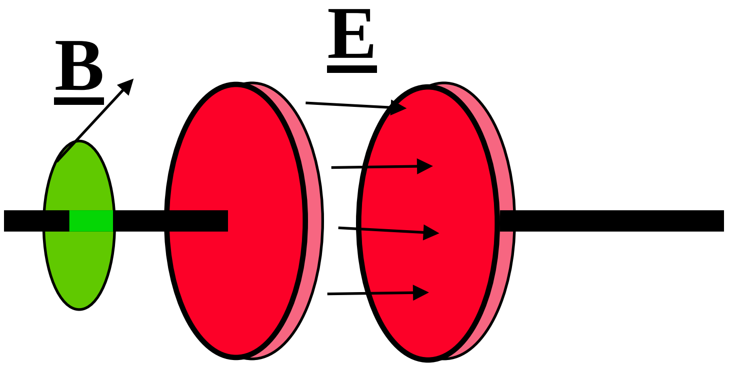
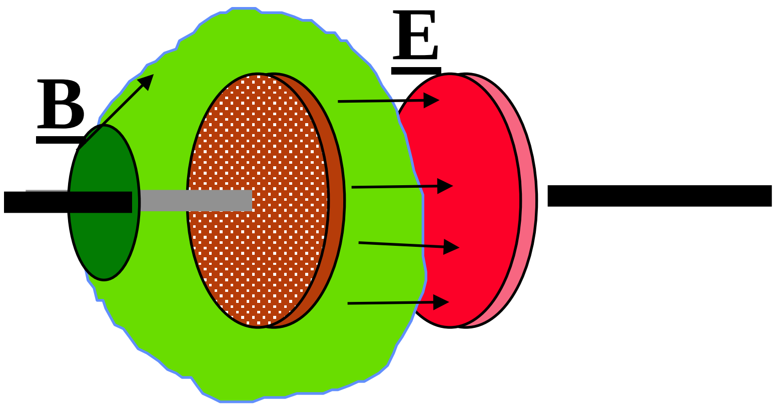
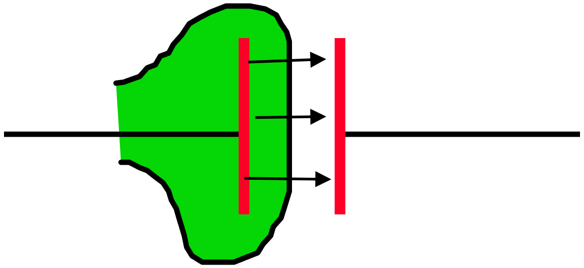



*.. if the loop encloses one plate of the capacitor..there is a problem … I = 0*

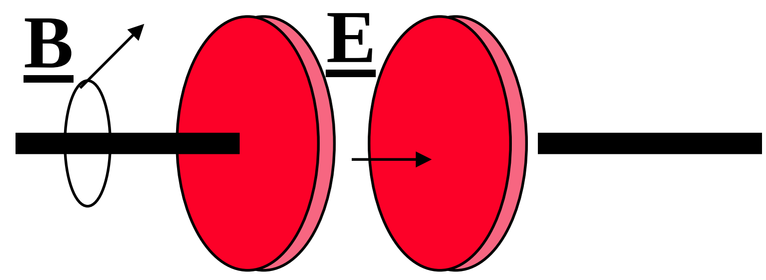



*Side view: (Surface is now like a bag:)*



# *Maxwell solved this problem by realizing that....*

**Inside the capacitor there must be an induced magnetic field...**




**How?.**

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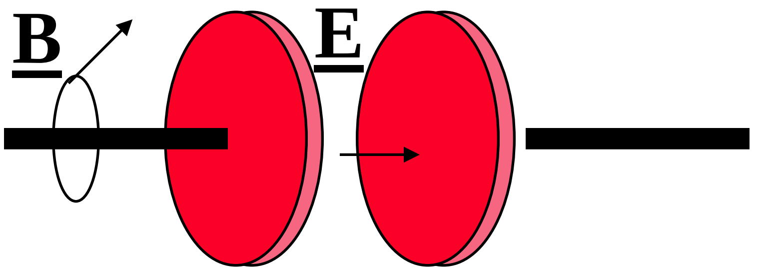
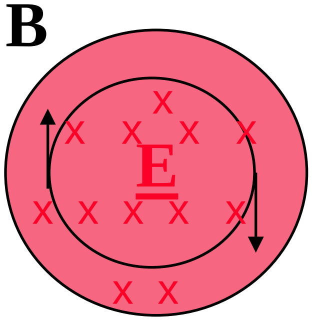



**How?. Inside the capacitor there is a changing E ⇒**




**A changing electric field induces a magnetic field**

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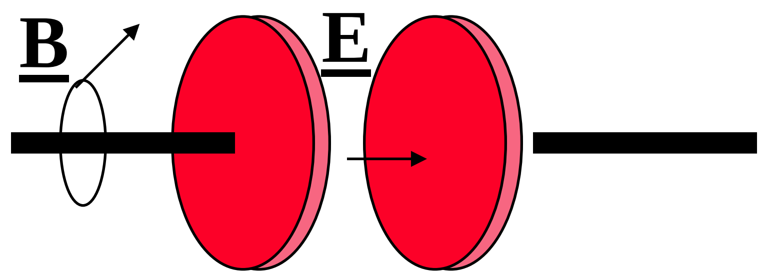


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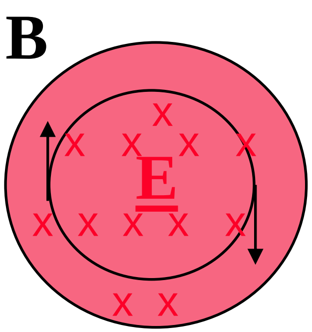


**A changing electric field induces a magnetic field**

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 I_d$$

***where $I_d$ is called the*** ***<u>displacement current</u>***

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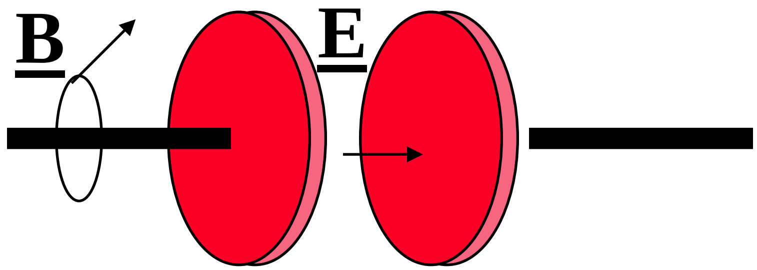


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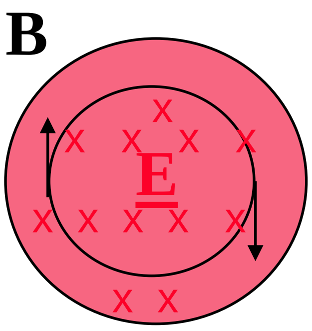


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***where $I_d$ is called the*** ***<u>displacement current</u>***

***Therefore, Maxwell's revision of Ampere's Law becomes....***

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

# Derivation of Displacement Current

For a capacitor, $q = \varepsilon_0 EA$ and $I = \dfrac{dq}{dt} = \varepsilon_0 \dfrac{d(EA)}{dt}$ .

Now, the electric flux is given by EA, so: $I = \varepsilon_0 \dfrac{d(\Phi_E)}{dt}$,
where this current , not being associated with charges, is called the "Displacement current", $I_d$.

Hence: $$I_d = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

# Derivation of Displacement Current

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Hence:
$$I_d = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

and:
$$\oint \underline{B} \bullet \underline{dl} = \mu_0 (I + I_d)$$

$$\Rightarrow \oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

# Maxwell's Equations of Electromagnetism

**Gauss' Law for Electrostatics**

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**Ampere's Law**

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

# Maxwell's Equations of Electromagnetism in Vacuum (no charges, no masses)

***Consider these equations in a vacuum.....***

***......no mass, no charges. no currents.....***

$$\oint \underline{E} \bullet \underline{dA} = \frac{q}{\varepsilon_0} \longrightarrow \oint \underline{E} \bullet \underline{dA} = 0$$

$$\oint \underline{B} \bullet \underline{dA} = 0 \longrightarrow \oint \underline{B} \bullet \underline{dA} = 0$$

$$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt} \longrightarrow \oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \longrightarrow \oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

# Maxwell's Equations of Electromagnetism in Vacuum (no charges, no masses)

$$\boxed{\oint \underline{E} \bullet \underline{dA} = 0}$$

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$$\boxed{\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}}$$

$$\boxed{\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}$$

# Electromagnetic Waves

***Faraday's law:*** **dB/dt ⟶ electric field**

***Maxwell's modification of Ampere's law***

**dE/dt ⟶ magnetic field**

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

$$\oint \underline{E} \bullet \underline{dl} = -\frac{d\Phi_B}{dt}$$

**These two equations can be solved simultaneously.**

**The result is:**

$$\underline{\mathbf{E}}(x, t) = E_P \sin(kx - \omega t)\ \hat{j}$$

$$\underline{\mathbf{B}}(x, t) = B_P \sin(kx - \omega t)\ \hat{z}$$

# Electromagnetic Waves

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

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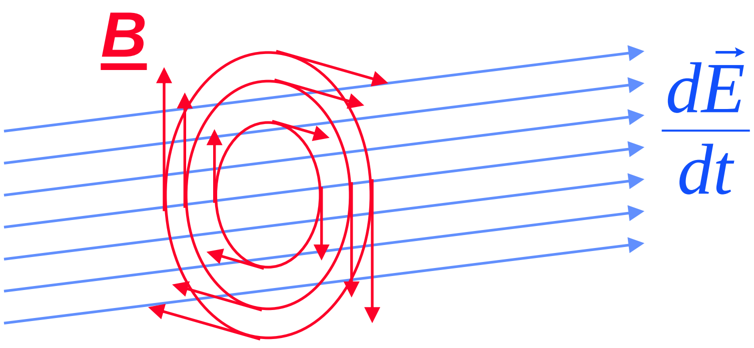


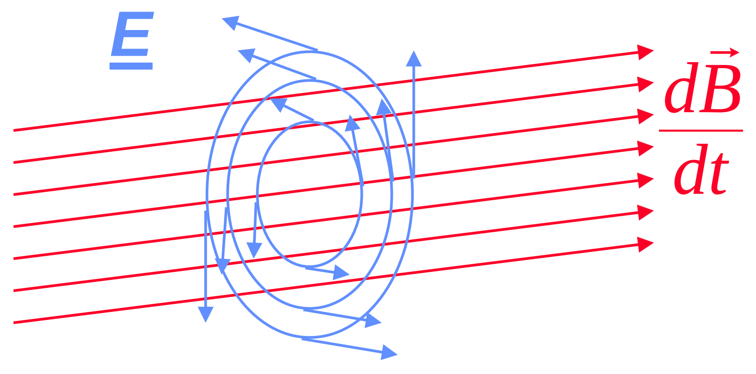

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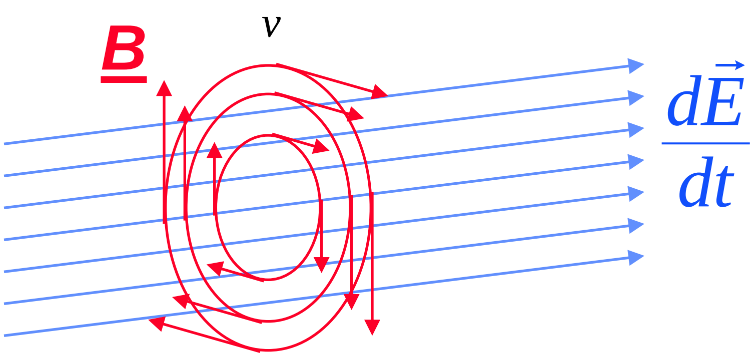
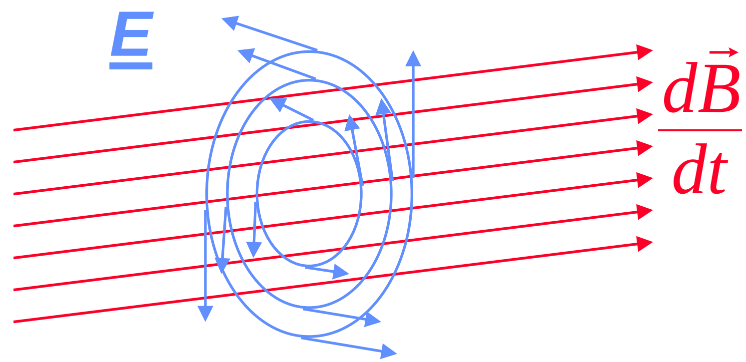


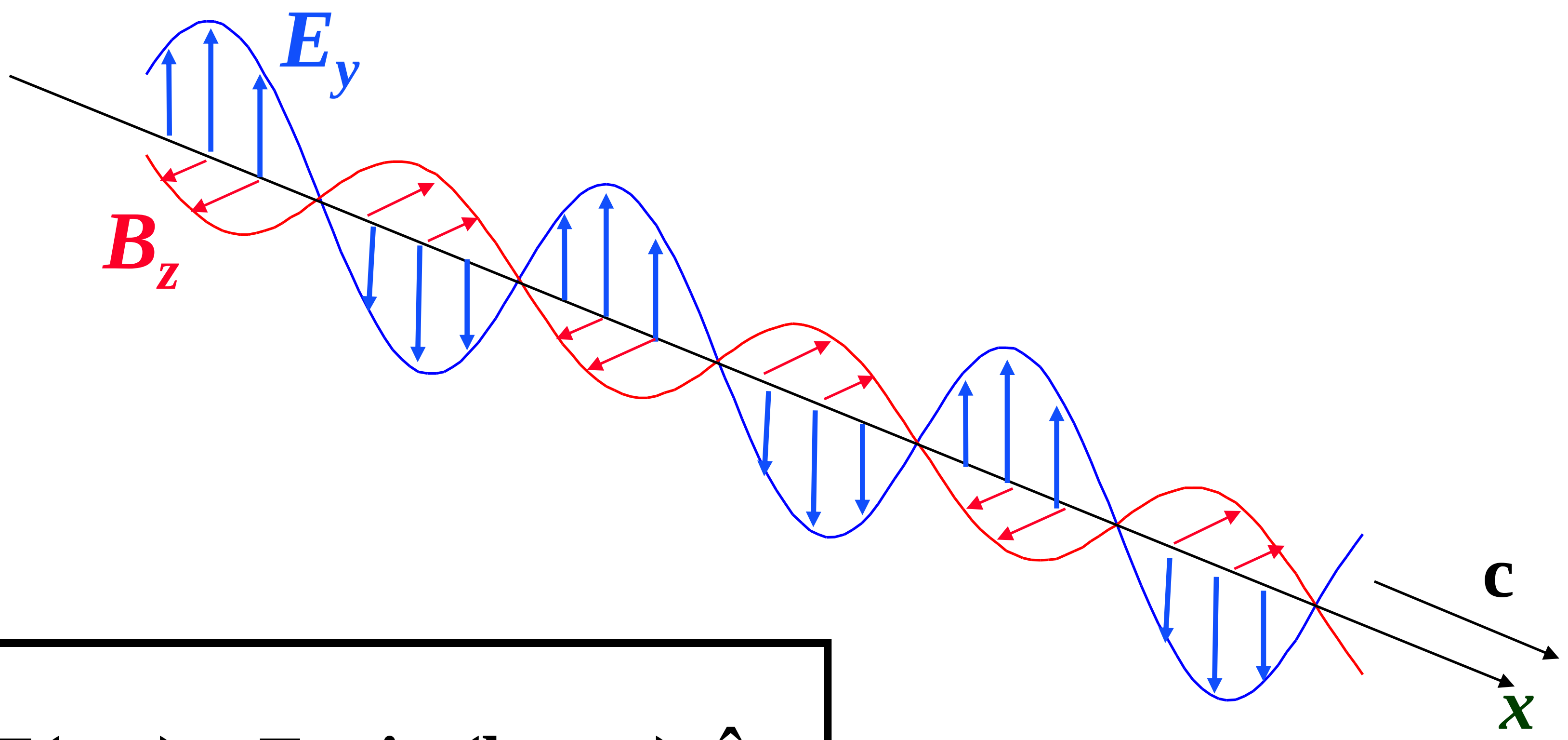

**Special case..PLANE WAVES...** $\vec{E} = E_y(x,t)\hat{j} \qquad \vec{B} = B_z(x,t)\hat{k}$

**satisfy the wave equation**

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

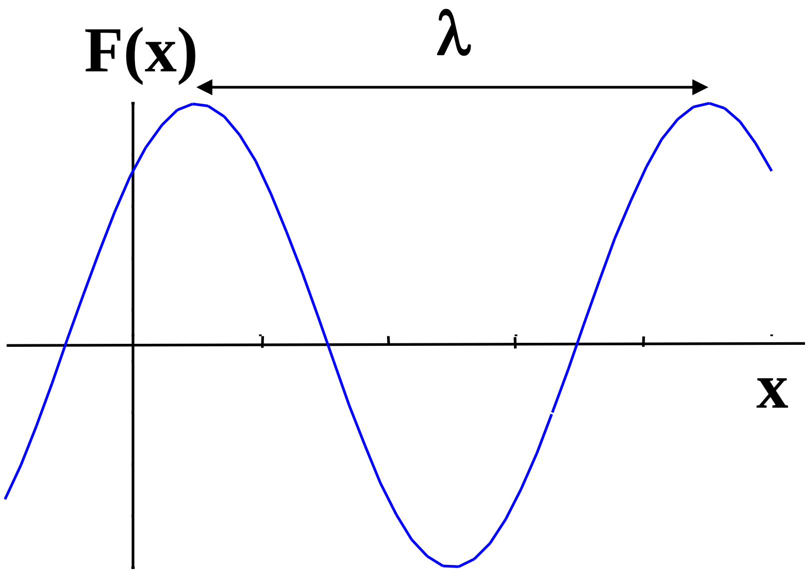
***Maxwell's Solution*** → $\boxed{\psi = A \sin(\omega t + \phi)}$

# Plane Electromagnetic Waves




$\underline{\mathbf{E}}(x, t) = E_P \sin(kx - \omega t)\ \hat{j}$

$\underline{\mathbf{B}}(x, t) = B_P \sin(kx - \omega t)\ \hat{z}$



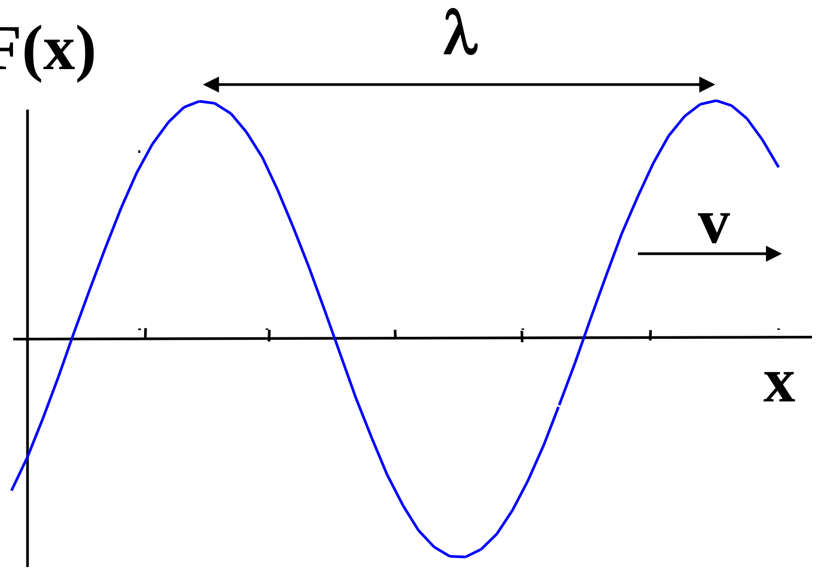

## Static wave

$F(x) = F_P \sin(kx + \phi)$

$k = 2\pi / \lambda$

$k$ = wavenumber

$\lambda$ = wavelength




## Moving wave

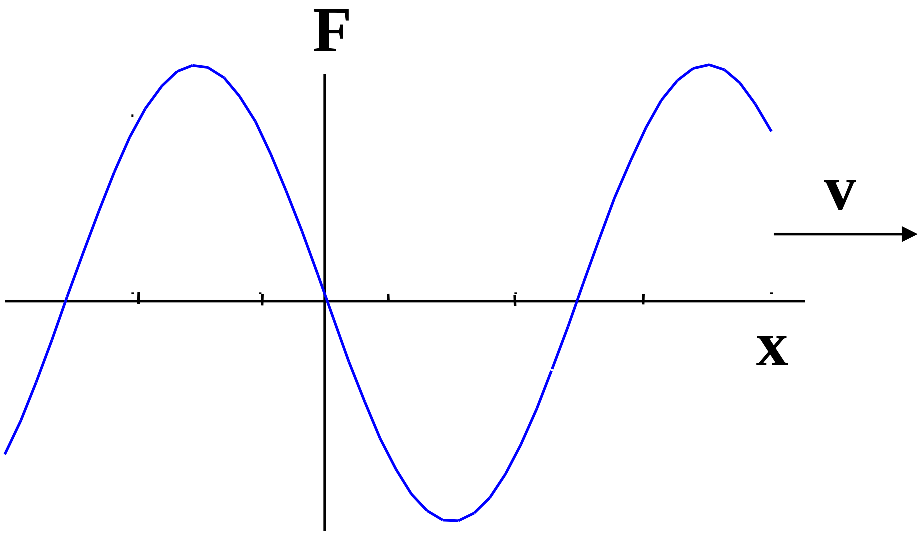
$F(x, t) = F_P \sin(kx - \omega t)$

$\omega = 2\pi / f$

$\omega$ = angular frequency

$f$ = frequency

$v = \omega / k$




# **Moving wave**

$$F(x, t) = F_P \sin (kx - \omega t)$$

What happens at x = 0 as a function of time?

$$F(0, t) = F_P \sin (-\omega t)$$

For $x = 0$ and $t = 0 \Rightarrow F(0, 0) = F_P \sin (0)$

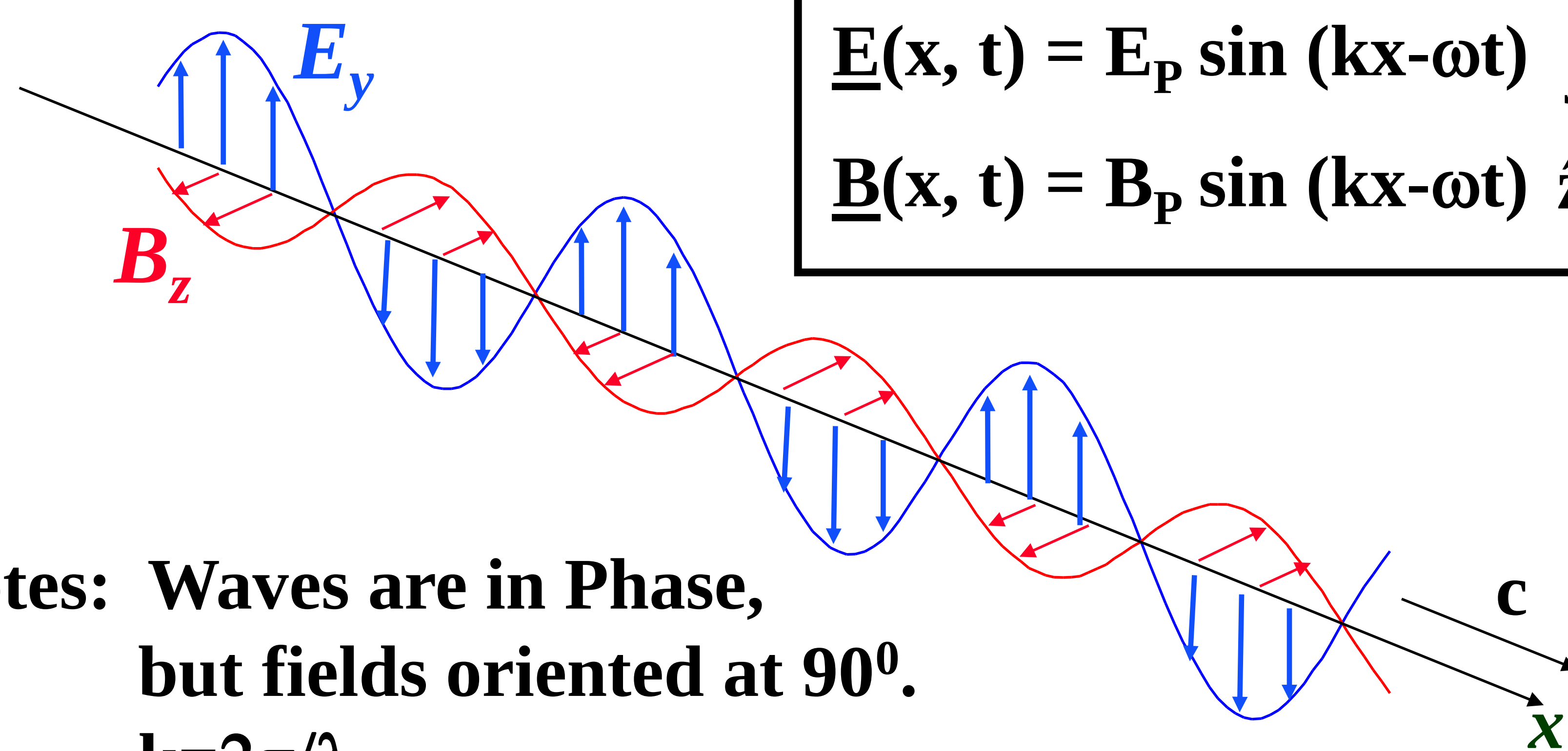
For $x = 0$ and $t = t \Rightarrow F(0, t) = F_P \sin (0 - \omega t) = F_P \sin (- \omega t)$

This is equivalent to: $kx = - \omega t \Rightarrow x = - (\omega/k)\, t$

**F(x=0) at time t is the same as F[x=-(ω/k)t] at time 0**

**The wave moves to the right with speed ω/k**

# Plane Electromagnetic Waves




$\underline{E}(x, t) = E_P \sin(kx - \omega t)\ \hat{j}$

$\underline{B}(x, t) = B_P \sin(kx - \omega t)\ \hat{z}$

**Notes: Waves are in Phase,**

**but fields oriented at $90^0$.**

**$k=2\pi/\lambda$.**

**Speed of wave is $c=\omega/k$ ($= f\lambda$)**

$$c = 1/\sqrt{\varepsilon_0 \mu_0} = 3\times 10^8 m/s$$

**At all times E=cB.**